Attachment to Telecom's Cost of Capital, Post-Workshop Submission

Comparing the tilted annuity and the building blocks approach to the risk free rate

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There is an important difference between the formulation of regulated cash flows based on the tilted annuity and the formulation of the regulated cash flows used by Dr Lally to conclude that the term of the risk free rate matches the regulatory period. The difference is that Dr Lally's argument allows the risk free rate to vary between regulatory periods whereas the tilted annuity does not. [1] The tilted annuity analytically requires that the risk free rate is constant over the expected economic life of the asset, as it is defined as geometric series with a constant ratio between successive terms. If the expected economic life of the regulated asset spans a number of regulatory periods then the term of risk free rate used by the tilted annuity is not necessarily the same as that for the risk free rate for any individual regulatory period.

Regulated cost based pricing for telecommunications services commonly relies on the tilted annuity method for determining depreciation plus the cost of capital of an investment. This method is an accepted regulatory practice and it is commonly used by regulators in other jurisdictions to determine the total service long-run incremental cost based price for interconnection and unbundled local loop services. In New Zealand, the Commission has used this method to determine the funding for the Telecommunications Service Obligation.

Regulated services in other industries such as gas and electricity networks use the *building blocks* approach to determine the allowed annual cash flow. It is within this context that Dr Lally bases his argument that the term of the risk free rate should be linked to the regulatory period.[2] Dr Lally takes the building blocks approach put forward by the ACCC in 1999 to prove that the term of the risk free rate should match the period of the regulatory price reset.

Comparing building blocks and tilted annuity bond rates

The difference between the building blocks and tilted annuity treatment of the risk free rate is illustrated by comparing of the *NPV=0 rule* under each approach. The Commission has established a regulatory objective of regulated prices satisfying the NPV=0 rule. This rule states that the sum of the expected net present value of all future cash flows, the initial outlay on the investment, and the value of options is to equal zero. The intention is to create the incentive for firms to invest while ensuring that prices are not above cost.

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[1] M. Lally, *Determining the Risk Free Rate for Regulated Companies*, prepared for the ACCC, August 2002, and M.Lally, *Regulation and the Choice of the Risk Free Rate*, April 2003
[2] Ibid.

In order to simplify the illustration of the features of interest, assume that the there is a regulated firm, prices are to be reset annually, and real options have no value. In addition, assume:

$V_0 =$ value of the firms initial capital outlay on asset to be regulated
$T =$ expected economic life of the investment
$t =$ period in years after the date of the initial outlay
$d_t =$ expected depreciation on the asset for period t at the start of period t
$a_{1,t} =$ one year bond rate for year t after the outlay. For $t > 0$ the value is not known.
$a_{n,t} =$ annualised n year bond rate for year t after the outlay. For $t > 0$ the value is not known.
$r_t =$ regulated revenue for year t

Under Dr Lally's approach to the building blocks method, the NPV=0 rule requires that the net present value of the future cash flow equals the initial capital outlay:

$$V_0 = \frac{d_1 + a_{1,1} \cdot V_0}{(1 + a_{1,1})} + \frac{d_2 + a_{1,2} \cdot (V_0 - d_1)}{(1 + a_{1,1})(1 + a_{1,2})} + \frac{d_3 + a_{1,3} \cdot (V_0 - d_1 - d_2)}{(1 + a_{1,1})(1 + a_{1,2})(1 + a_{1,3})} \cdots$$
$$\cdots + \frac{d_T + a_{1,T} \cdot (V_0 - d_1 - d_2 \ldots - d_{T-1})}{(1 + a_{1,1})(1 + a_{1,2})(1 + a_{1,3}) \ldots (1 + a_{1,T})} \qquad (1)$$

This is the net present value of the regulated cash flows taken over the full life of the asset.

In contrast, the tilted annuity formulation of the NPV=0 rule is:

$$V_0 = \frac{X_1}{(1+a)} + \frac{X_1(1+g)}{(1+a)^2} + \frac{X_1(1+g)^2}{(1+a)^3} \cdots + \frac{X_1(1+g)^{T-1}}{(1+a)^T} \qquad (2)$$

Where a is the expected return over the life of the asset, and g is the expected change in the replacement cost of the regulated asset - ie, $V_1 = V_0(1+g), \quad V_2 = V_0(1+g)^2$, ... etc.

The problem here is to determine a value of X_1, where X_1 is the sum of the depreciation and cost of capital in the first year. Summing the geometric series and solving for X_1 gives:

$$X_1 = \frac{V_0(a-g)}{1 - \frac{(1+g)^T}{(1+a)^T}}$$

The general annuity equation cited in the Commission reports is the annuity for period t. It is derived from the tilted annuity geometric series and sum, resulting in the expression:

$$X_t = \frac{V_0(a-g)(1+g)^{t-1}}{1-\frac{(1+g)^T}{(1+a)^T}}$$

The tilted annuity models the effect of competition on cash flows. It assumes that the effect of competition on cash flows is proportional to the change in the cost of building a competing network, which is captured by the factor $(1+g)$. If $g=0$ then there is no change in cost over time; if $g>0$ then cost is increasing; and, if $g<0$ then cost is decreasing. In sum, the NPV=0 rule under (2) determines the regulated revenue by explicitly matching the expected depreciation (or appreciation) to the expected change in value of a new replacement investment.

However, the value of X_t cannot be derived analytically from the building blocks formulation represented by (1). This is because the bond rates used in (1) are related to the regulatory period, and these rates are very likely to differ from one regulatory period to the next.

Determining the tilted annuity expected return

The debate regarding the selection of a risk free rate for calculating the cost of capital has been framed as, first, selecting the appropriate term of the risk free rate, and then selecting the government bond that matches this term, (where, the government bond acts as a proxy for the risk free rate). This methodology produces the correct result when applying the building blocks approach described by (1) as the term of the risk free rate directly relates to each period in a series of one-off regulatory decisions. However, this methodology violates the NPV=0 rule when applied to the tilted annuity approach described by (2) since the pattern of the regulatory cash flow described by a tilted annuity traverses a series of regulatory decisions.

It is proposed here that rather than inferring a risk free rate based on the term of the regulatory cycle, that the risk free rate for a tilted annuity is calculated based on the weighted average of the 1 year, 2 year, 3 year .. etc bond rate, (whether observed or estimated from the yield curve), and where the weights are determined by the change in the replacement cost of the regulated asset base. That is, start by calculating the value of the expression:

$$\frac{V_0}{X_1} = \frac{1}{(1+a_{1,0})} + \frac{(1+g)}{(1+a_{2,0})^2} + \frac{(1+g)^2}{(1+a_{3,0})^3} \ldots + \frac{(1+g)^{T-1}}{(1+a_{T,0})^T} \qquad (3)$$

The term to the right of the equals sign can be evaluated based on the shape of the yield curve and expected change in the replacement cost of the asset. The value of a then can be numerically solved for by substituting the value of (3) into the expression, derived from (2):

$$\frac{V_0}{X_1} = \frac{1}{(1+a)} + \frac{(1+g)}{(1+a)^2} + \frac{(1+g)^2}{(1+a)^3} \ldots + \frac{(1+g)^{T-1}}{(1+a)^T}$$

The intuition behind this interpretation of a is based on the observation that the titled annuity specifies the regulated cash flow over the expected economic life of the asset. As the name implies it can be thought of as an investment that provides an annuity with a tilted cash flow that lasts for the life of the investment. In contrast to the tilted cash flow pattern of the annuity, a government bond has a different cash flow pattern which includes coupons and the principle. Therefore, (3) constructs the tilted annuity as a series on one-off zero-coupon bonds, which is based on the yield of government bonds, where the term of the zero-coupon bond corresponds to the date of the annuity.

Using this methodology ensures that the tilted annuity expected return does not violate the NPV=0 rule, and avoids the difficulties created by the arbitrary application of the building blocks approach in this special context.