A Dynamic Model of Next-Generation Networks with Two-Sided Platform Competition

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Abstract

This paper presents a dynamic model of competition between two IP-platforms - also known as Next-Generation Networks (NGN). We discuss several technological elements and economic aspects that have emerged from the literature concerned with NGN. First, a NGN serves multiple types of consumers, so we consider end-users and content providers who both require connection to a network platform. Second, we make some assumptions about traffic flows (i.e. who originates a transmission and whether a consumer or content provider gets charged or not). Third, different interconnection agreements are considered, that range from regulated access charges through zero-charge agreements.

Our model is distinguished by four main components: (1) we model utility functions for both end-users and content providers; such utility functions then determine the demand functions for two services: voice and data; (2) market shares are determined as a function of a switching parameter that determines the inflow of customers to one platform from the other as they compete over prices; (3) on-net and off-net traffic patterns are then determined as a function of market shares; (4) platforms compete non-cooperatively and profits depend on the realization of each of the above. The dynamics of competition are modeled by letting the model run for a number of time periods. At each period t, equilibrium prices are obtained; equilibrium prices are a function of market shares and content provision in period t - 1 along with prices and traffic patterns in period t.

We first use the model to examine some traditional issues involving access pricing for voice telecommunications services and transit versus peering arrangements for data services. Simulation is then organized around three additional criteria that include (1) whether platforms charge content providers with subscription or usage fees, (2) whether platforms discriminate between on-net and off-net data pricing for end-users and content providers, and (3) whether platforms charge or block off-platform content. The combinations of results from systematically applying these criteria provides a rich variety of scenarios which we use to describe the dynamics of usage prices, subscription fees, market shares, platform profits, content provider profit, consumer surplus, and total social welfare.

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1 Introduction

This paper presents a dynamic model of competition between two IP-platforms - also known as Next-Generation Networks (NGN). We discuss several technological elements and economic aspects that have emerged from the literature concerned with NGN[2]. The model is implemented in a computational tool that facilitates the analysis of equilibrium in the market in a sequence of time periods, which allows a realistic and manageable methodology to represent the dynamics of competition in the market.

The model is based on the work developed by De Bijl and Peitz (2002), but it introduces significant modifications that are needed to model the technological characteristics of NGN as well as other market and regulatory assumptions that are appropriate in an IP environment. The baseline model is a model of competition between two operators, each serving two types of consumers. The model aims to describe the results of competition between the two operators through prices, market shares, profit levels, consumers' and overall welfare. The pricing structure used by the operators is a two-part tariff: subscription charge and usage charge. The dynamics of competition is observed and measured at discrete periods of time and this framework allows us to observe the evolution of key variables in response to changes in regulated access charges and strategic decisions by the operators.

2 Literature Review

Our results extend the work of De Bijl and Peitz (2002)[3]. Their study deals with two networks that compete on the telephone service market. Its objective is to gain insight into the nature of competition and entry in the phase towards mature competition, and to understand the effects of regulatory instruments on market structure, consumers' surplus and incentives for investment. The analysis is based on the design of policy and regulatory measures to promote competition in markets for fixed voice telephony. De Bijl and Peitz examine three types of entry: facilities-based competition, direct access through "local loop unbundling", and indirect access through "Carrier Select." In all studied situations, there is a market dominated by an incumbent. Retail price and wholesale prices are the key determinants that unleash the dynamics of competition. The analysis focuses on comparing how regulation affects an entrant's profits in different entry scenarios, and the entrant's incentives to invest in infrastructure.

A significant body of research on network competition has been recently produced. The first important contribution to the literature on network competition was made by Laffont, Rey and Tirole in two papers

[2] Although key technological issues are considered and discussed, in this paper we are not modeling specific issues of control, access or security.
[3] This research was commissioned by Opta, the telecommunications services authority in the Netherlands.

(1998a, 1998b), which develop a model of completely unregulated competition between two interconnected networks. A number of different pricing scenarios is considered, including linear pricing, two-part tariffs, and in the second of the two papers price discrimination between on-net and off-net traffic. Wholesale access prices for the termination of traffic that originates on one network and terminates on the other, plays an important role in the analysis, and the authors consider both access prices set by a regulator and prices negotiated by the networks themselves. Their work concludes that a competitive (Nash) equilibrium can fail to exist if access prices are sufficiently greater than the cost of termination and the networks are viewed by consumers as close substitutes. Assuming an equilibrium exists, a benevolent regulator would set access charges below marginal cost in order to maximize total surplus, while collusive networks would like to set access charges greater than marginal cost in order to implement the profit maximizing final price resulting from competitive interaction. The authors also show how price discrimination can create a new source of network externalities, since the customers of a given network gain or lose as other customers subscribe according to whether the on-net price is less than or greater than the off-net price. This price differential has ambiguous effects. The authors, however, show that under linear pricing only a unique, shared-market equilibrium is stable. Since under non-linear pricing price discrimination imposes a cost on any markup over the cost of access prices, firms who negotiate access prices would be expected to agree to reciprocal access charges equal to marginal cost.

A number of papers have appeared more recently with the objective of modeling platform competition in two-sided markets. See, for example, Rochet and Tirole (2003), Armstrong (2003), Schmalensee and Evans (2007) and the references therein. These papers seek to model a varied set of industries including banking services, financial exchanges, software platforms, shopping malls and dating clubs. The common element in each such industry is the fact that a platform attracts two (or more) sets of customers who seek to interact with one another. A key variable in explaining the evolution and efficiency properties of two-sided platforms is the ability of their pricing policies, involving both fixed and variable charges, to overcome a fundamental network externality that end users are unable to resolve by themselves.

Another series of recent research papers has been concerned with interconnection of telecommunications networks, when it is assumed that both senders and receivers of messages derive benefits from calls. See, for example, Atkinson and Barnekov (2000), DeGraba (2003), Hermalin and Katz (2006), and Jeon, et al. (2004). Like the papers on two-sided platforms, these papers are concerned with the ability of networks, and in this context regulatory policy, to internalize the externalities that are inherent in two or multi-party communications environment. Degraba and Hermalin-Katz focus on usage-based externalities and the ability of networks to allocate the cost of a call between the sender and receiver efficiently. Atkinson and Barnekov are primarily concerned with the subscription externality and the role which interconnection policies play in determining a customer's decision to subscribe to a network, and in the competition between networks for subscribers.

In a paper closely related to the present one, Laffont, Marcus, Rey and Tirole (2003) specifically consider a model of internet interconnection. Their model consists of a two-sided platform in which consumers, on one side, request data traffic from data service providers, such as ISPs on the other side. It is assumed that end-user consumers originate traffic only to data service providers, and that data service providers only send traffic to consumers upon request. Therefore, on a given platform, the outgoing traffic from each node is entirely due to customer requests for data downloads from service providers, and the incoming traffic to each node is due to data supplied to consumers upon request from the data service providers. A terminating access price is set, either by negotiation or by regulation. Whenever the data service provider and the customer are on different networks, the customer network collects revenue equal to the access price from the network to which the data service provider subscribed.

3 Technological Elements and Economic Aspects of IP Networks

As the transition towards fully converged IP networks will take some time, with intermediate stages depicting a hybrid network until transition is complete, our model attempts to represent some of the transitional elements that appear to be most relevant (ITU 2007), (Cohen 2007). From a network architecture perspective, NGNs will eliminate the hierarchy of existing telephone networks and will reduce the number of nodes in the network.

Who pays for the call?

One feature to consider is the answer to “who is paying for the call?” Under "Calling Party Pays" pricing, a caller initiating a phone call pays for the entire traffic sensitive cost of the call. This form of pricing could be fully efficient if the caller receives all the benefit of a two party voice communication, but it has been widely noted that alternative arrangements might lead to superior outcomes in cases where both the call originator and the call recipient benefit from communication. Exceptions to calling party pays pricing are toll free numbers (X-800), whereby the receiver of the call pays the full marginal cost. In an IP-based communication, it is important to distinguish between call originator (in the case of voice over IP or VoIP, for example) and the party transmitting traffic packets. In spite of the fact that whoever originates a call can be clearly identified, the network is used as much by the receiver's packets as the caller's packets. In mobile networks in the U.S. and some other countries, receivers as well as originators of calls are routinely charged. It is likely that similar pricing arrangements will exist in future IP networks (Marcus 2006).

Interconnection agreements

Interconnection agreements among networks serving Internet Service Providers (ISPs) are of two types: peering and transit. When two ISPs sign for a peering agreement, no charges are incurred by any of the involved parties for the exchanged packets, and these packets cannot travel beyond the receiver's network onto a third party. On the other hand, an interconnection agreement may involve a client-provider agreement, whereby one party, the client, pays for access to the provider's network. In this case, the provider has the responsibility to deliver the client's traffic to every destination, by acting as a transit network[4].

Network neutrality

The network neutrality principle requires non-discrimination against particular sites on the Internet and devices used to access the Internet.[5] The concept has become very controversial, evolving to also mean that a network operator must not discriminate among contents traveling on its network. In particular, network operators may be prohibited from giving priority for a particular type of traffic, especially traffic whose content is generated by its own subscribers or affiliates. Another way to understand network neutrality is to say that of all the participants in the value chain, only the end users should be charged for access and use. In this interpretation, content providers, applications and software should not pay for their traffic on a broadband network.

Network effects

There are two types of network effects identified in the literature: direct and indirect (Leibowiz and Margolis). Direct network effects are those attributed to the non-mediated effect of the number of purchasers on the value of a product. Indirect network effects are those where complementary products become more attractive (availability or price) as the number of consumers of a product increases.

What are the effects that a new user joining a platform produces on other platform users' perceived value? An end-user benefits from the presence of other users on the network. Having access to more potential

[4] An early attempt to model these problems is developed by Yoon (2006).
[5] The concept of "network neutrality" is credited to Professor Tim Wu (Wu, 2003).

receivers of his traffic is generally regarded as a positive effect on a user. The two types of consumers considered in this model may perceive the effect differently.

An end-user consumer benefits when he or she has greater access to content providers. He also benefits from being able to reach other end-users either through using an e-mail service or P2P application. In fact, a free, downloaded P2P application is very attractive to most consumers. We assume that an end-user's value for the network increases as the number of either type of users increases.

A large number of users may indeed lead to periods of congestion in the network. Congestion has an effect on the user that is perceived as negative. However, it is the use of the network that creates congestion and not the mere number of users. If a network has a sufficiently large number of "heavy" users, that is users who consume a lot of network resources, the network may experience congestion, and the effect would be detrimental to a user's benefits. In the simulations to be described below, we will assume that network operators are able to provide sufficient network capacity and differential pricing (which we do not model) so that congestion is not a significant problem to users.

Content providers, or websites competing for end-users' business, also may benefit from access to large numbers of potential customers connected to the provider's network. However, content providers would normally perceive no benefit in subscribing to a network containing many other content providers, and potentially there could be a negative network effect due to increased competition for their services.

4 A Model of Competition

As telephone networks evolve towards fully IP networks, the distinction among consumers becomes more evident. For example, on an IP network websites' users can have access to information and services; on the other hand, ISPs, who also subscribe to the network, can sign up individual consumers or companies to provide them with Internet access, hosting and other services. Therefore the network is regarded here as a two-sided market, that is, a platform that attends to different types of consumers.

In this section we introduce a facilities-based competition model. We regard a network operator as a platform that attracts different types of consumers who connect to the network, purchasing services from the platform. This section describes a dynamic model of competition between two platforms. The market is two-sided in the sense that platforms compete for both retail consumer subscribers and for content providers who serve the consumers.

At time zero, prices are set by each platform. For every new time period the model computes the platforms' profits, and calculates a Nash equilibrium based on market shares and content levels attained in the previous time period as well as prices set in the current period. A more detailed description of each of the model components is presented below.

4.1 Platforms

Two platforms, platform 1 and platform 2, compete in a market that has two types of consumers: content providers and end-users. At time zero, platform 1 (the incumbent) has a very large market share. Except for the initial market share, networks are symmetrical. That is, their structure, reach and cost characteristics are assumed to be similar, though not necessarily identical.[6]

Platform 1 and platform 2 compete under a two-part tariff scheme. We will distinguish between two services: voice and data. Platform i's initial market shares in those markets are $\alpha_i(0)$ and $\beta_i(0)$ for voice and data, respectively. Platforms provide services to both content providers and end-users, charging them prices in accordance to the following notation: (for *i* or *j* = 1, 2)

p_{ij} = price to consumers of platform *i* for voice traffic with another consumer on platform *j*

q_{ij} = price to consumers of platform *i* for data traffic with a content provider on platform *j*

r_{ij} = price to content providers on platform *i* for data traffic with a consumer on platform *j*

m_i = subscription price for consumers on platform *i*

n_i = subscription price for content providers on platform *i*

We assume that if content providers are charged a usage price, it applies only when the provider terminates data traffic on the platform imposing the charge. Operators' market shares in both markets are a direct function of the number of subscribers of each type. Initial conditions are exogenously imposed to set platform market shares in period 0.

4.2 Demand for Platform Services

In a two-sided platform we need to distinguish between end-users' perceptions and content providers' perceptions about the network or the service being consumed. In particular the benefit each derives from transacting over the platform depends on their particular economic characteristics.

[6] In our default input assumptions and all of the simulations described below, we assume that the relevant cost characteristics of the two platforms, other than initial market share, are in fact identical.

Consumers derive utility from sending messages, x_{Voice}, to other consumers (on either platform) and from receiving content, x_{Data}, from network content providers (on either platform). The utility from content is assumed to depend on the total content made available to a platform as well as the number of minutes of data services consumed. Consumer utility is assumed to be separable between voice messages and data services. Individual consumer's demand functions for voice and data services, $x_{Voice}(p)$ and $x_{Data}(q, Content)$ are illustrated in Figure 1.

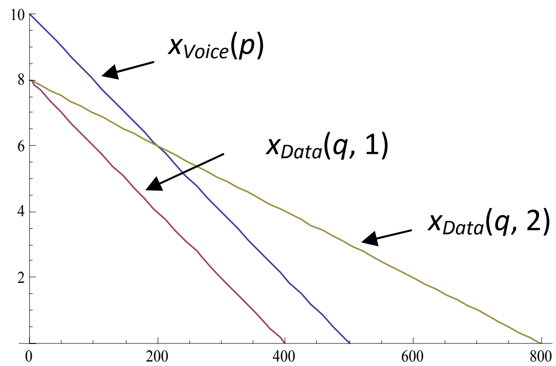


Figure 1: Demand for Voice and Data Services

Content is assumed to be supplied in an increasing marginal cost production environment. There may be fixed costs which give rise to traditional economies of scale. The total cost function is:

$$TC(Content) = \Pr oviderFixedCost + \frac{\Pr oviderVariableCost\ Content^2}{2} \quad (1)$$

We assume that content provision is competitively supplied. Hence, a content provider will supply content up to the level at which price is equal to marginal cost. Therefore, a content provider facing a market price equal to ρ will supply content up to the point at which marginal cost is equal to ρ. That is,

$$ContentSupply(\rho) = \frac{\rho}{\Pr oviderVariableCost} \quad (2)$$

Content providers earn revenues from advertising. Advertising revenue is assumed to be equal to a constant fraction (AdRevMultiplier) of total consumer utility per unit of data output. When

- the platform's customer market share is α,
- one unit of content is supplied, and,
- customer data price is equal to q

it follows that

$$Ad\,\mathrm{Re}v(q,\alpha) = \alpha\ Ad\,\mathrm{Re}vMultiplier\ ContentUtility(x_{Data}, 1) \quad (3)$$

Content providers may pay usage fees to platform operators. Total fees paid by a provider offering one unit of content on a platform with customer market share α, where the provider charge per data minute is equal to *r* and customer charge per data minute is equal to *q,* are given as follows:

$$UsageFee(q,r,\alpha) = r\alpha\, x_{Data}(q,1) \tag{4}$$

The "price" ρ that content providers face in choosing its level of content is therefore equal to the advertising revenues minus usage fees paid to network platform operators for each unit of content. As a function of relevant platform prices and market shares, using (3) and (4), content supply of a provider on platform *i* is given by

$$\begin{aligned} ContentSupply(q_{1i}, q_{2i}, r_{1i}, r_{2i}, \alpha_1, \alpha_2) = ContentSupply\big[&Ad\,\mathrm{Re}v(q_{1i}, \alpha_1) + Ad\,\mathrm{Re}v(q_{2i}, \alpha_2) - \\ &UsageFee(q_{1i}, r_{1i}, \alpha_1) - UsageFee(q_{2i}, r_{2i}, \alpha_2)\big] \end{aligned} \tag{5}$$

Both advertising revenues and usage fees depend on the market share of end-user consumers on each platform. Each of the relevant prices in the above function is determined as a Nash equilibrium at the conclusion of every time period.

The network externality functions generalize and replace the fixed utility functions that played an important role in the de Bijl and Peitz analysis. In the current framework, consumer fixed utility and content provider fixed profit may depend on a particular platform's market share. Since entrants initially have a small market share relative to the incumbent, the entrant operates with a competitive disadvantage initially. Over time, if market share grows, entrants and incumbents compete on a more equitable basis [7].

ConsumerFixedUtility represents the value of a communications network that is not captured by voice and data usage (e.g. the option value of receiving calls, making emergency calls, or having access to the internet). In addition, the fixed utility can represent the way that an average consumer evaluates the quality of a particular platform, which depends on the number of other consumers who subscribe to that platform. Finally, a consumer may value a platform having many content providers more highly because of higher quality data transfers or possible blocking of content from providers on another platform. The ConsumerExternality function captures all of these effects.

ProviderExternality represents the value that suppliers have to be on the same network with consumers plus the value that suppliers have to share the same network with other suppliers. The last function is

[7] The network externality functions generalize and replace the fixed utility functions that played an important role in the De Bijl and Peitz analysis. In that analysis, the authors assumed that entrants are initially handicapped because consumers (and in our model, content providers) could expect to receive only a fraction of fixed utility or profit in early time periods. This fraction was assumed to increase linearly to 1 over a finite number of periods.

generally zero or negative because as more vendors join the same network, competition grows leading to lower profits.

The following functions represent the externality functions, where all parameters C_{X1}, C_{X2}, P_{X1} and P_{X2} are assumed to be nonnegative, and values near to zero represent minimal externalities.

$$
\begin{aligned}
ConsumerExternality\left(\alpha_i,\beta_i\right) &= \frac{\left(1+C_{X1}\alpha_i\right)\left(1+C_{X2}\beta_i\right)}{\left(1+C_{X1}\right)\left(1+C_{X2}\right)} ConsumerFixedUtility \\
ProviderExternality\left(\alpha_i,\beta_i\right) &= \frac{\left(1+P_{X1}\alpha_i\right)\left(1-P_{X2}\beta_i\right)}{\left(1+P_{X1}\right)\left(1-P_{X2}\right)} ProviderFixedUtility
\end{aligned} \tag{6}
$$

Finally, the consumer indirect utility $V_i(\ )$ and provider profit functions $P_i(\ )$ are used to determine dynamic market share functions and to define consumer and content provider surplus. These functions incorporate a "balanced traffic" assumption, which means that a given consumer on either platform is equally likely to place a voice call to any other consumer on either platform, and is equally likely to request a data transfer from any other content provider on either platform. We will discuss this assumption further below where platform profit functions are explicitly computed. Indirect utility for a consumer on platform i facing usage prices p_{i1} and p_{i2} for voice traffic and q_{i1} and q_{i2} for data traffic from platforms 1 and 2 respectively, and a subscription price m is given as follows.

$$
\begin{aligned}
V_i\left(p_{i1},p_{i2},q_{i1},q_{i2},m_i,t\right) = {} & ConsumerExternality\left[\alpha_i(t-1),\beta_i(t-1)\right] + \\
& \alpha_1(t-1)\, VoiceUtility\left(x_{Voice}(p_{i1})\right) + \alpha_2(t-1)\, VoiceUtility\left(x_{Voice}(p_{i2})\right) + \\
& \beta_1(t-1)\ ContentUtility\left[x_{Data}\left(q_{i1}, Content_1(t-1)\right), Content_1(t-1)\right] + \\
& \beta_2(t-1)\ ContentUtility\left[x_{Data}\left(q_{i2}, Content_2(t-1)\right), Content_2(t-1)\right] - \\
& \alpha_1(t-1)\ p_{i1}x_{Voice}\left(p_{i1}\right) - \alpha_2(t-1)\ p_{i2}x_{Voice}\left(p_{i2}\right) - \beta_1(t-1)\ q_{i1}\ x_{Data}\left(q_{i1}, Content_1(t-1)\right) - \\
& \beta_2(t-1)\ q_{i2}\ x_{Data}\left(q_{i2}, Content_2(t-1)\right) - m_i
\end{aligned} \tag{7}
$$

Note that both the content functions (defined above) and the market share functions (defined below) are defined with a one period lag. In what follows we will drop the explicit notational dependence of the indirect utility function on current period prices (p_{i1}, p_{i2}, q_{i1}, q_{i2}, m_i) and instead will write $V_i(t)$ to represent $V_i(p_{i1}, p_{i2}, q_{i1}, q_{i2}, m_i, t)$.

Profit functions for a content provider connected to either operator are functions of consumer usage prices and provider prices for on-net and off-net data service. By convention, we assume that content providers benefit from both on-net and (potentially from) off-net advertising revenues, and are charged only when they deliver content to a customer on a particular platform. Again, dropping the dependence of the provider profit function on prices (in this case q_{i1}, q_{i2}, r_{i1}, r_{i2}, n_i) for notational convenience, we can express the profit function for a provider connected to platform *i* as:

$$\begin{aligned} P_i(t) = & \Pr oviderExternality[\alpha_i(t-1), \beta_i(t-1)] + \\ & \{Ad\operatorname{Re}v(q_{1i}, \alpha_1(t-1)) + Ad\operatorname{Re}v(q_{2i}, \alpha_2(t-1)) - \\ & UsageFee(q_{1i}, r_{1i}, \alpha_1(t-1)) - UsageFee(q_{2i}, r_{2i}, \alpha_2(t-1))\}\, Content_i(t-1) \end{aligned} \tag{8}$$

4.3 Market Shares

$\alpha_i(t)$ and $\beta_i(t)$ represent platform *i*'s market share for consumers and content providers subscribed to it in any period *t*. Market shares are arbitrarily determined at the start (period 0) and updated for every subsequent period.

Current market share, at time *t*, depends on the differential utility or profit that a consumer or provider can expect on each platform, as well as the market share at *t*-1 and the competitive intensity parameters, Z_C and Z_P, which measure a consumer's loyalty to a platform. A higher value of Z_C may indicate that the platform is more able to retain its customers perhaps through some type of locked-in service offers; it also may signal that customers feel some kind of loyalty to the platform so they become "hard to switch". A higher value of Z_P, the provider intensity parameter, may similarly indicate that providers are less likely to switch to the other platform due to the existence of already convenient commercial agreements between providers and platform. Also, technical disruptions in their service may force providers to think twice before switching to another platform.

We assume that customer loyalty is a value that is uniformly distributed on the interval [0, Z_C] for end-users and [0, Z_P] for content providers. A fraction of end -user consumers and content providers are assumed to switch from platform 2 to platform 1 whenever the difference between the utility (profit) that they would derive from platform 2 is larger than the utility they currently derive from platform 1. The fraction of consumers or providers who choose to switch increases as the difference in utility or profit increases. In the simulations, we assume that current market shares for platform 1 are determined as follows:

$$\alpha_1(t) = \alpha_1(t-1) + \frac{V_1(t) - V_2(t)}{Z_C \ \alpha_1(t-1)}$$

$$\beta_1(t) = \beta_1(t-1) + \frac{P_1(t) - P_2(t)}{Z_P \ \beta_1(t-1)} \tag{9}$$

We assume that all consumers and content providers choose to participate in the market by choosing either platform 1 or platform 2.[8] Accordingly, $\alpha_2(t) = 1 - \alpha_1(t);\ \beta_2(t) = 1 - \beta_1(t)$. Since the market share functions $\alpha_i(t)$ depends on consumer utility functions on V_1 and V_2, they are necessarily a function of the entire set of prices p_{ij},, q_{ij}, r_{ij}, m_i, and n_i (i, j = 1,2) as well as t. Each $\beta_i(t)$ is a function of the same set of prices in each period t.

Equations (9) describe an important property of the current model, and the way in which it differs from a traditional product differentiation model. In a product differentiation approach, consumers are assumed to have continuously varying, but fixed, preferences for each of two firms. In the classic Hotelling model, consumers are located along a road segment with identical firms at each end point. Customers care about transportation costs (i.e. distance from each firm), and competition between the firms occurs in a single period as the firms set prices in order to segment the market. In equilibrium, the marginal consumer, and only this consumer, is indifferent between patronizing one firm or the other.

In the present approach, competition occurs over multiple periods. As equations (9) show, market shares are expected to change in every period in which the indirect utility of consumers, or the profit of content providers differs between the two competing firms at their current prices. When one of the firms is an entrant, the interpretation is that entry (or potentially exit) occurs in early periods as the firms dynamically adjust their prices. Market shares continue to adjust up to the point at which every consumer and every content provider is indifferent between the firms. Beyond this point, further entry or exit ceases. Competition then enters a stable phase in which the market for consumers and content providers is segmented in much the same way as in a product differentiation model.

4.4 Costs

Platform operators incur both variable and fixed costs. Variable costs on platform i for the origination of voice traffic (including transport) destined to the other platform are given by c_{i1} and d_{i1}. For incoming traffic, the marginal costs of terminating voice and data traffic (including transport) are given by c_{i2} and d_{i2}. We assume that the costs of customer requests for data traffic (origination and termination) are negligible (i.e. for data traffic $c_{i1} = c_{i2} = 0$). Marginal "on-net" costs for voice and data traffic are given respectively by $c_{13} = c_{11} + c_{12}$

[8] This assumption also requires that fixed utility and fixed profit parameters have been specified appropriately.

and $d_{13} = d_{11} + d_{12}$. Fixed costs of network operators are given by f_C, representing the fixed cost of attaching an additional voice subscriber to a platform, and f_P, the fixed cost of attaching an additional content provider.

Platform operators also incur costs in the form of access charges for terminating calls which their own subscribers originate, and which terminate on the opposing network. In the case of voice calls, access charges are traditionally set by a regulator, and in our notation they are denoted by ta_{1Voice} and ta_{2Voice} respectively for platforms 1 and 2. For data services, termination charges between networks are traditionally set by negotiations between platforms. Under peering arrangements, these charges are set equal to zero, while under transit arrangements a positive termination charges is agreed upon by both platforms. In either case, we assume that data termination charges are set independently of the competitive dynamics which determine all retail prices charged directly to consumers or content providers. The wholesale termination charges for data services will be denoted by ta_{1Data} and ta_{2Data}.

4.5 Platform Profit Functions

From a platform's viewpoint, both voice and data traffic can be on-net, off-net or incoming. More specifically:

- On-net traffic: it is voice or data traffic originating and terminating on a single platform.
- Off-net traffic: it is voice traffic that originates from an on-platform subscriber and terminates with another platform's subscriber, and data traffic requested by an on-platform subscriber from a content provider on another platform.
- Incoming traffic: it is voice traffic that originates from a subscriber on another platform and terminates with an on-platform subscriber, and data traffic requested by a customer from another platform to an on-platform content provider.

It is useful to be specific about the components of the calling patterns as they will be used in the calculation of revenues for each platform. The general principle to be used in the calculation of traffic patterns is that whenever an end-user initiates any communication, the receiving consumer (end-user or content provider) may be any other consumer with equal probability, regardless of the platform he or she is connected to. This "balanced traffic" assumption has been used by Laffont, Rey and Tirole (1998a) and virtually all of the subsequent literature on network competition. The following diagrams illustrate the assumed traffic patterns on Platform 1 for both voice and data traffic with associated marginal costs.

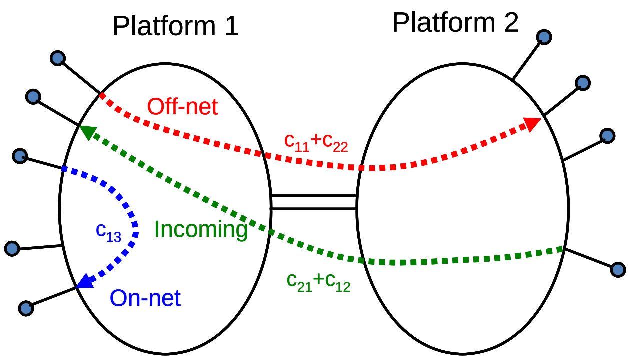


Figure 2: Voice Traffic Patterns

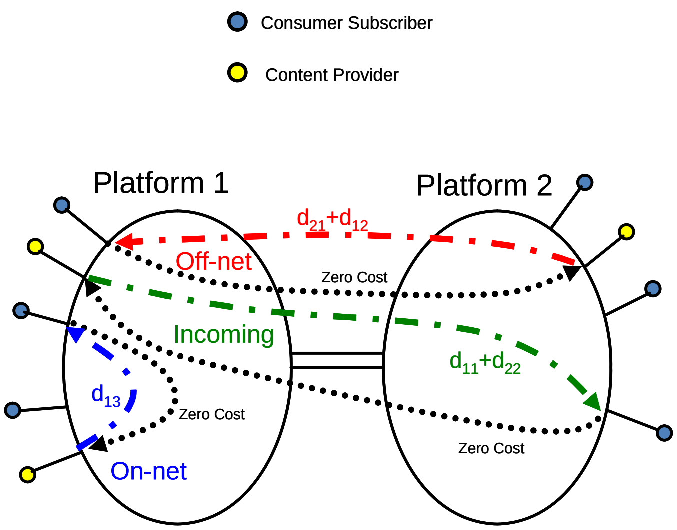


Figure 3: Data Traffic Patterns

Platform operators are able to charge both customer subscribers and content providers usage fees per unit of output and fixed charges. Since each platform's profit depends on its market share of both end-user consumers and content providers, profit in each period is a function of all retail prices set by both platforms. A A platform's profit function is obtained adding up te contributions to profits from on-net, off-net and incoming traffic plus the revenue accrued by the subscriptions.

5 Solving the Model

5.1 First Order Conditions

Platforms are assumed to compete non-cooperatively in every time period using subsets of the full set of retail prices specified above. The full computation loop to solve the model is shown in the flowchart of Figure 4 and explained in the following paragraphs.

Nash equilibria can then be determined by solving the following set of first order conditions, keeping in mind the dependence of each profit function on the full set of prices:

$$\frac{\partial \pi_k(t)}{\partial v} = 0 \text{ with } v \in \{p_{11}, p_{12}, p_{21}, p_{22}, q_{11}, q_{12}, q_{21}, q_{22}, r_{11}, r_{12}, r_{21}, r_{22}, m_1, m_2, n_1, n_2,\} \text{ and } k \in \{1,2\} \qquad (10)$$

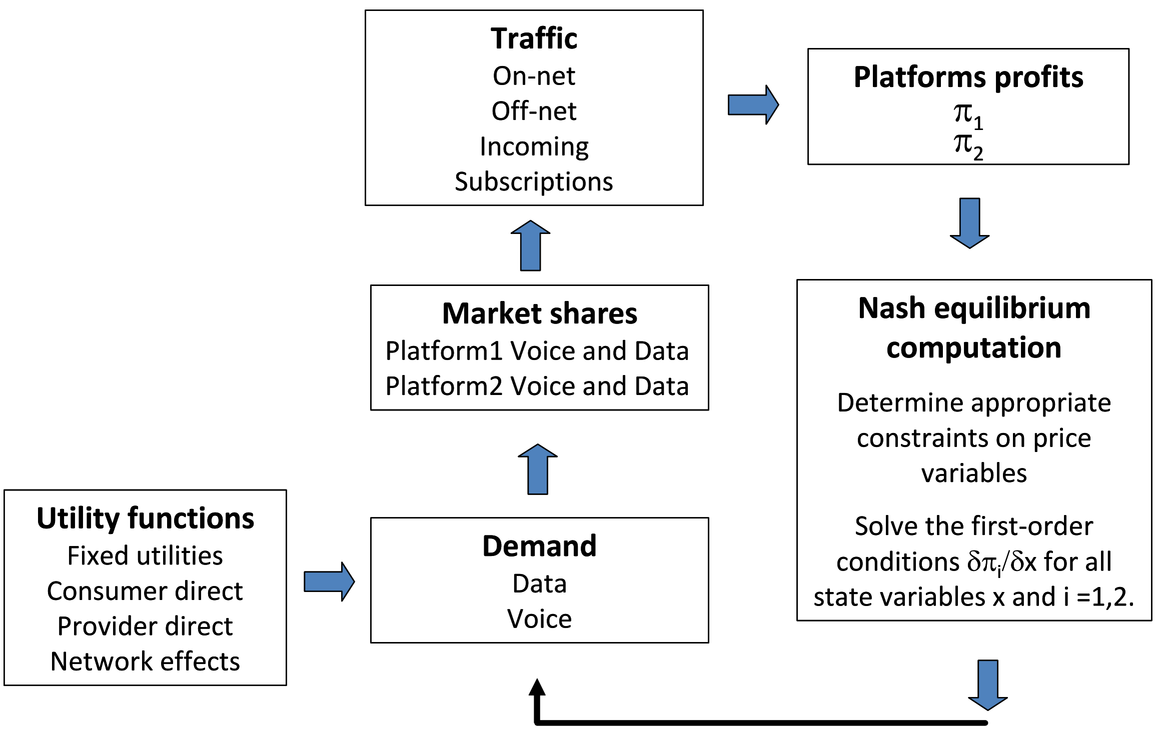


Figure 4. Computation loop

Platforms do not necessarily compete using the full set of price variables described above. Subscription prices and usage prices for content providers may or may not be available to platform operators; platforms may or may not discriminate between on-net and off-net data pricing for either consumers or content providers; and finally platforms may or may not charge off-platform content. Therefore, one or more constraints (e.g. $n_1 = n_2 = 0$) are typically imposed and the constrained system is then solved. A solution is obtained by first specifying an arbitrary set of initial starting values for all price variables, market shares and content levels in period 0. Using Newton's method as implemented in Mathematica™, we find a solution for all prices, market shares and content levels in period 1. These values are used as starting values for an identical optimization in period 1, and the process continues for as many periods as desired.

Given the equilibrium prices, the current period market shares for both consumers and content providers are obtained. Simultaneously, these prices and the volume of traffic determined by the market share function determine the profits for each platform. Market shares and provider content on each platform are then used as inputs in the calculation of next period equilibrium prices.

5.2 Perceived Marginal Costs for Consumer Services

In previous work on network competition, it is a standard result that when firms are able to charge both a customer subscription fee and a customer usage fee, usage charges must be set equal to the perceived marginal cost facing each firm[9]. In this case firms use the subscription charge to compete for market share and the usage charge to optimally generate call volume. If there is no price discrimination between on-net and off-net services, perceived marginal cost depends on traffic proportions between on-net and off-net traffic, which by the balanced traffic assumption depends on relative market shares. For voice traffic, perceived marginal cost on platforms 1 and 2 is given as follows:

$$\begin{aligned} \mathrm{PMC}_{1Voice}(t) &= \alpha_1(t)\, c_{13} + \alpha_2(t)\left(c_{11} + ta_{2Voice}\right) \\ \mathrm{PMC}_{2Voice}(t) &= \alpha_2(t)\, c_{23} + \alpha_1(t)\left(c_{21} + ta_{1Voice}\right) \end{aligned} \tag{11}$$

For customer data traffic without price discrimination, similar results hold as given by the following expression.

$$\begin{aligned} \mathrm{PMC}_{1Data}(t) &= \beta_1(t)\, d_{13} + \beta_2(t)\left(d_{12} - ta_{1Data}\right) \\ \mathrm{PMC}_{2Data}(t) &= \beta_2(t)\, d_{23} + \beta_1(t)\left(d_{22} - ta_{2Data}\right) \end{aligned} \tag{12}$$

If price discrimination is possible, then the perceived marginal cost of platform 1 for on-net voice traffic is equal to the marginal cost of originating and terminating traffic on the same platform (c_{13}). For off-net voice traffic on platform i, the perceived marginal cost is equal to the marginal cost of origination plus the terminating access charge on the rival platform ($c_{i1} + ta_{jVoice}$). Similar remarks apply to the perceived marginal cost of data traffic assuming price discrimination.

[9] See, for example, Laffont and Tirole (2000, p. 207).

In a closely related model of network competition, Laffont, Rey and Tirole (1998a, Proposition 7, 1998b, Proposition 5) demonstrated that under certain conditions a unique symmetric equilibrium exists both with and without price discrimination. The sufficient conditions in both results required that either (i) terminating access price is close to marginal cost, or (ii) competition is sufficiently weak, which in the product differentiation model assumed implies that the networks are sufficiently poor substitutes.[10]

De Bijl and Peitz (2002, Proposition 1) extend the results of Laffont, Rey and Tirole to a model like the present one in which firms are symmetric, but compete in an environment with customer switching costs. In this context, existence and uniqueness of equilibrium can be shown if either (i) terminating access prices are close to marginal cost, or (ii) the competitive intensity parameters are sufficiently large.

5.3 Usage Prices for Content Providers

If usage fees for content providers are feasible, it is immediately apparent that it is necessary to constrain the fee imposed on off-platform content in order to avoid unstable outcomes. For example, if platform 1 is allowed to raise r_{12} without constraint, it could simultaneously increase its own profit and lower the profits of a content provider on platform 2, which would have the effect of immediately reducing the market share of providers on platform 2. Accordingly, we will assume that if usage fees are possible, then either $r_{ij} = 0$, or that $r_{ij} = r_{ii}$ for $i \neq j$.

Next, note that if subscription fees for content providers are feasible, then platform operators have no reason to impose in addition a positive or negative usage fee on providers. Platform profits can be set at any desired level using the subscription fee, and do not depend on current period values of provider content.

If usage fees are feasible and subscription fees are not feasible, new issues arise. Since consumers, rather than content providers, determine the total volume of data services in our formulation, usage fees can potentially create an unstable and self-destructive form of competition between platform operators. The latter can be explained with an example. Suppose that platforms do not set different prices for on and off platform services and that both on and off platform content providers can be charged a usage fee. Therefore, if $r_{11} = r_{12}$ and $q_{11} = q_{12}$, it follows from equations (8) that $Content_1$ (t - 1) = $Content_2$ (t -1). Keeping in mind that all functions of current period t are also functions of the complete set of prices, while market shares and content levels in period $t-1$ are constants, it can be proved that

[10] In Bertrand duopoly models with perfect substitutes, non-existence of pure strategy equilibrium is the expected result except in cases where there are no fixed costs and where neither competitor has a cost advantage. The significance of the results of Laffont et al. is to precisely characterize the conditions under which these restrictive conditions can be relaxed in order to allow for the existence (and uniqueness) of equilibrium.

$$\frac{\partial \pi_1(t)}{\partial r_{11}} = \alpha_1(t)\,\beta_1(t)\,xData(q_{11}, Content_1(t-1))$$

The latter shows that as long as platform 1 has a positive market share in both consumer and provider markets, and $xData(q_{11}, Content_1\,(t\text{-}1)) > 0$, its profits are increasing in r_{11}. Since a similar result holds for platform 2, there can be no solution to the first order conditions (10) under these conditions.

The previous results suggest a platform has incentives to strategically set usage prices to its advantage. In other words, we conjecture that platforms would still find it profitable to increase the usage charges for providers up to the point at which content supply is driven to zero; the latter still needs to be formally proved. Nevertheless, assuming our conjecture is valid, its implication is that consumer demand for data services would also fall to zero, with a consequent collapse of the market for those services. In order to gain some insights into the possible consequences of allowing platforms to charge usage fees to content providers, we assumed in our simulations that such fees are determined on a contractual basis, outside of the normal competitive process.[11]

6 Results

6.1 Representative Outputs for a Base Case with Default Inputs

This section presents the set of outputs for a **base case** using the default set of input prices and parameters as documented in Appendix 1. The base case assumes that platforms are able to set both a usage fee and a fixed subscription fee to customer subscribers and platforms charge a fixed fee but no usage fee to content providers. The default demand function for voice traffic is similar to the demand specification used by De Bijl and Peitz, who argued that their specification roughly approximates the demand for voice traffic in the Netherlands[12]. Our results should therefore be interpreted as shedding insight only on the qualitative changes in consumer surplus, content provider profit and network platform profits as various values of the input parameters change.

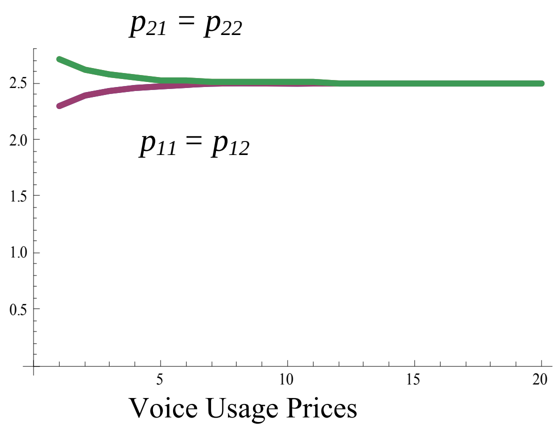
The default inputs also roughly correspond to access and transit pricing agreements that are in place today in some markets. That is, the terminating access charge for voice traffic is assumed to be greater than the

[11] If a platform operator is allowed to charge usage fees only to content providers on its own platform, it is possible that such usage charges could be determined competitively through the first order conditions (10). However, we have not yet been able to find parameter values for the model that confirm that such equilibria exist.

[12] Otherwise, no attempt has been made at present to calibrate either the demand for data traffic or the relevant cost parameters with values appropriate for next generation networks.

marginal cost of termination, while the terminating access charge for data traffic is assumed to be equal to zero.

Figures 5 through 8 illustrate the dynamic paths of relevant price, market share and surplus functions over the time horizon studied, which in all cases we assume is equal to 20 periods. In the default inputs, a period is assumed to consist of a single calendar quarter, which is based on an assumption that network platforms compete by adjusting prices four times in every calendar year.[13] The simulations therefore show the dynamic effects of competition over a five-year time horizon.



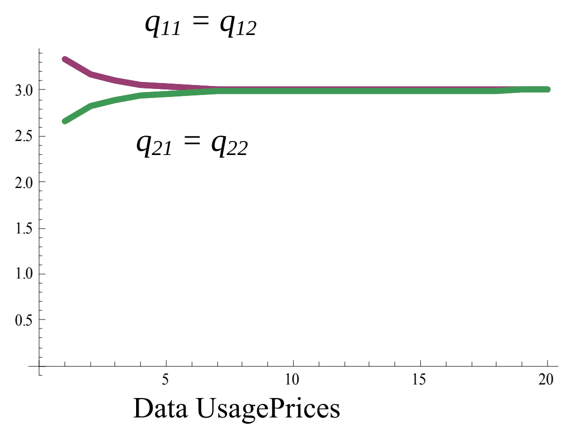
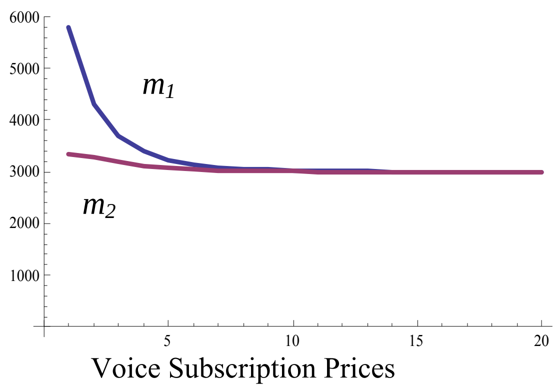


Figure 5: Consumer Voice and Data Prices



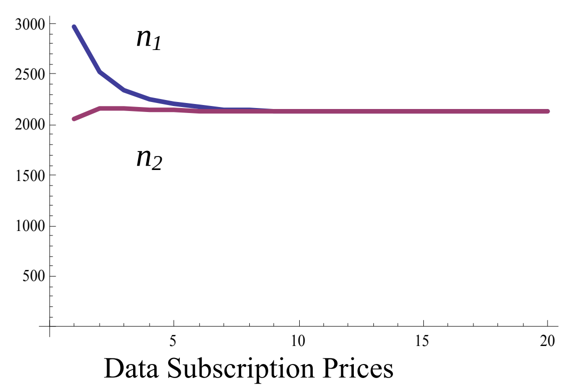
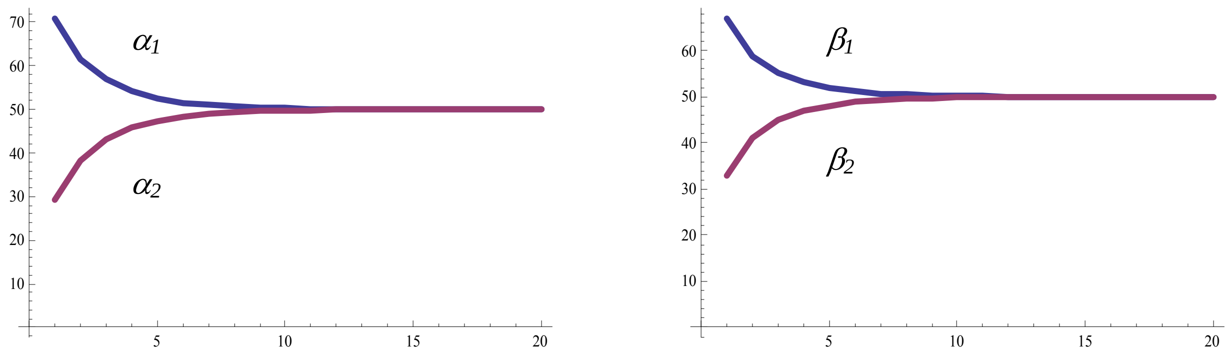


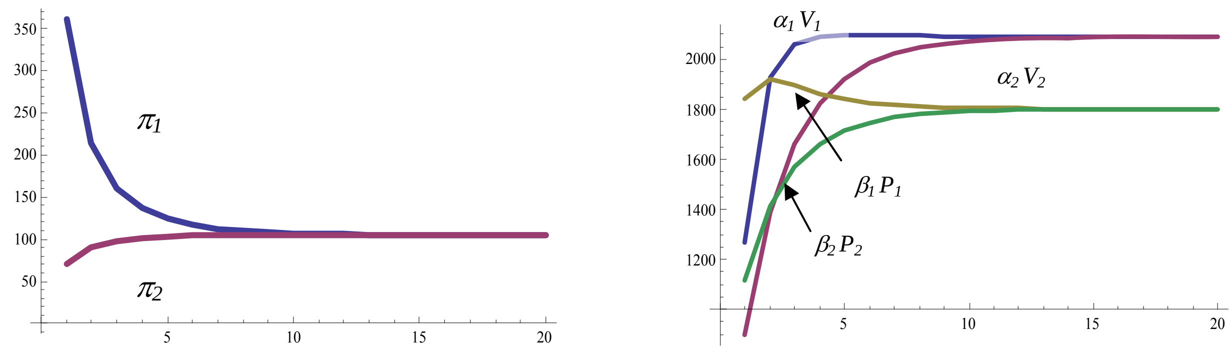
Figure 6: Consumer and Content Provider Subscription Fees

[13] The discount rate used to summarize total surplus and profit results in all future computations is appropriately adjusted for the length of period. In the default set of inputs, we assume an annual interest rate of 10%, which implies a per period discount factor of 0.976



Voice Market Share Data Market Share

Figure 7: Market Shares in Consumer and Content Provider Markets



Platform Profits Consumer and Producer Surplus

Figure 8: Operator Profits and Surplus by Type

6.2 Alternative Values of the Competitive Intensity and Network Externality Factors

Among the most significant input parameters, in terms of determining both competitive dynamics and final summary performance measures are the competitive intensity parameters, Z_C and Z_P, and the market externality factors C_{X1}, C_{X2}, P_{X1}, and P_{X2}. In this section, we briefly summarize how our base case results may be expected to change when these input parameters are modified.

First we consider different values Z_C and Z_P. As noted above, these roughly correspond to transportation costs in a traditional Hotelling differentiated product model. However, a Hotelling model is designed to segment a linear market between customers (and in our case content providers), who choose platforms 1 or 2 immediately in every period. In contrast, the dynamic model of De Bijl and Pietz assumes that only a fraction of the customers who could potentially benefit from switching from one platform to another actually do so in any time period. The size of this fraction is directly and inversely related to the value of Z_C or Z_P. Thus, the higher these values, the fewer the number of customers (or providers) who will choose to switch platforms in any given time period, in response to a potentially attractive increment in consumer surplus or content provider profit. In Figure 9 below, we show the results of choosing higher switching cost parameters for both consumers and content providers. Here we assume that Z_C is 50% greater than its default value, while Z_P is 66% larger than its default value. Figure 9 differs from the comparable Figure 8 primarily in the higher levels of platform profits and the lower levels of consumer and producer surplus for subscribers to each platform.

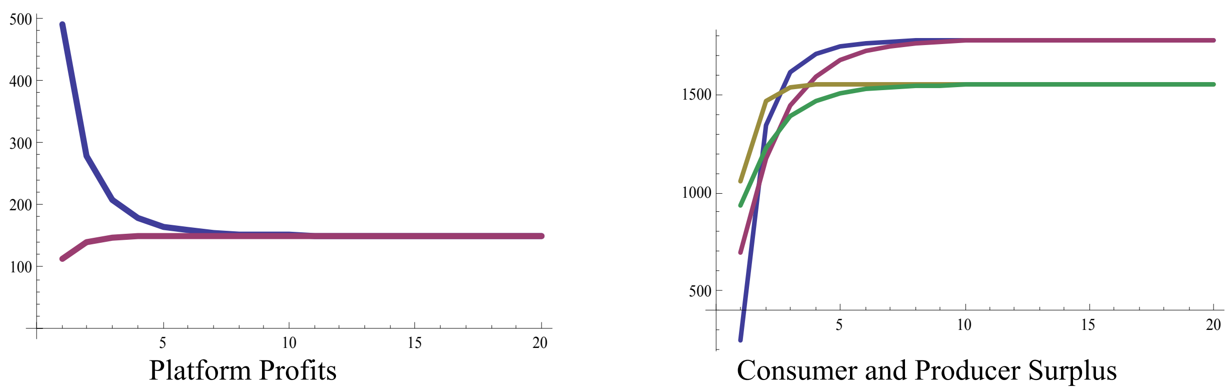
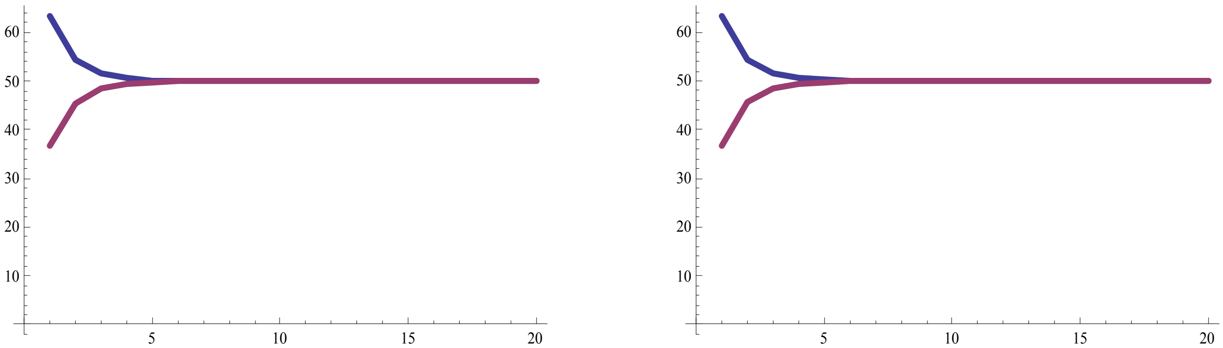


Figure 9: Operator Profits and Surplus with Higher Switching Costs

It is also relevant to modify the network externality parameters. As equation (9) makes clear, whenever the input parameters C_{X1}, C_{X2}, P_{X1} or P_{X2} are positive, the Consumer Externality function and the Provider Externality function will both potentially depend on market shares α_i and β_i for either platform *i*. When these parameters are equal to zero, both consumers and providers receive a common fixed utility or profit in every period which does not depend on market share. The primary difference between models with and without network externalities is that as externalities decline, competition becomes more intense, since the initial incumbent advantage due to large market share is negated. Figure 10 shows the dynamic paths of both consumer and content provider market share in a simulation in which each of the externality factors is assumed to be equal to zero. Table 3 in the following section shows the quantitative impact of these changes.



Voice Market Share   Data Market Share

Figure 10: Market Shares in Consumer and Content Provider Markets With Zero Externalities

6.3 Monopoly and Ramsey Benchmarks for the Base Case

While most of our results so far are concerned with the dynamic model of network competition it is useful to also consider two relevant benchmarks against which future results can be compared. To define the monopoly benchmark, we assume that the entire market is served by a single firm (platform 1) for each of the 20 periods. All traffic is by definition on-net, and we assume that the monopoly firm sets both usage prices and fixed fees in every period so as to maximize platform profits, subject to the constraints that consumers and content providers will choose to participate in the market – i.e. $V_1(t) \geq 0$ and $P_1(t) \geq 0$ for every t.

To define a "Ramsey" benchmark, we assume that two firms equally share the market in every time period, and that all prices are set so as to maximize total surplus, subject to the constraints that consumers, content providers and platforms are all willing to participate – i.e. $V_i(t) \geq 0$, $P_i(t) \geq 0$ and $\pi_i(t) \geq 0$ for $i = 1, 2$.[14] Table 1 shows both monopoly and Ramsey benchmark results the competition measures of interest.

Table 1: Benchmark Surplus and Profit with Default Inputs

	Total Surplus	**Consumer Surplus**	**Provider Profit**	**Platform Profit**
Monopoly	15482	0	0	15482
Ramsey	13391	6234	7157	0
Competition Default Inputs	13105	4951	4437	3717
Competition (Z_C = 10000 Z_P = 5000)	13101	4111	3759	5230

[14] Since platforms are able to charge fixed subscription fees to consumers in all cases, and to content providers in some of our scenarios, there is, in general, no unique set of prices (including subscription fees) that will maximize total surplus. The Ramsey benchmark results reported below therefore assume that platform profits are constrained to be exactly equal to zero.

Table 2 shows the set of usage prices and fixed fees that generate these results, assuming that there is no price discrimination between on-net and off-net services. Note in particular that the equality of consumer prices and perceived marginal costs for consumer services no longer holds for the profit maximizing and total surplus maximizing benchmarks. For voice services, benchmark prices are equal to the true marginal costs (equal to c_{i3} for on-net voice service and $c_{i1} + c_{j2}$ for off-net service), and for data services, benchmark prices are below the true marginal costs (equal to d_{i3} for on-net data traffic and $d_{i2} + d_{j1}$ for off-net traffic).[15] The latter result holds because both customer subscribers and content providers benefit from an increase in data traffic, while customer subscribers alone determine the volume of that traffic. It is therefore efficient to price data services below marginal cost, and recover the incremental lost profits through higher fixed charges for customers or (if possible) for content providers.

Table 2: Benchmark Prices with Default Inputs

	p	*q*	*r*	*m*	*n*
Monopoly	2.0	1.83	0	9486	6855
Ramsey	2.0	1.83	0	3584	446
Competition Default Inputs	2.5	3.0	0	3000	2125
Competition (Z_C = 10000 Z_P = 5000)	2.5	3.0	0	3625	2625

Since the fixed utility of consumers and the fixed profit of content providers can depend on market share, monopoly has an inherent advantage in attracting both consumers and providers when externality factors are positive. Hence total surplus under monopoly is higher than the Ramsey total surplus in these cases. When network externalities are absent, total surplus is the same under both monopoly and the Ramsey calculation which maximizes total surplus. The distribution of this surplus is, of course significantly different, as the following table shows.

Table 3: Benchmark Surplus and Profit (Zero Network Externality Parameters**)**

	Total Surplus	**Consumer Surplus**	**Provider Profit**	**Platform Profit**
Monopoly	14784	0	0	14784
Ramsey	14784	7829	6955	0
Competition Default Inputs	14462	6224	4665	3573
Competition (Z_C = 10000 Z_P = 5000)	14462	5373	3984	5105

[15] In our default inputs, the marginal cost of both on-net and off-net voice traffic is equal to 2, and the marginal cost of both on-net and off-net data traffic is equal to 4. It is only by accident (given default assumptions about termination charges) that the perceived marginal cost for data services is exactly equal to the true marginal cost.

7 Concluding Comments

In this paper we have presented a model of network competition that could be an appropriate tool for the analysis of competition in the telecommunications industry as that industry evolves from one centered on voice communications services to one in which consumers, independent firms offering information services and network operators all interact using next-generation networks as their fundamental infrastructure. Our results confirm results found by previous studies of network competition, and offer a number of new insights regarding the interrelationship between wholesale pricing of voice and data.

Our analysis is clearly preliminary in nature. In the remainder of this section we offer a few brief comments on both the limitations of the current framework and some possible suggestions for future research. First, and most obviously, we have created a model with a large number of input parameters which can all potentially be important in determining final results. Necessarily, our analysis has had to focus on a small number of possible parameter changes and interactions among these parameters. It would be desirable to examine more carefully the way in which our results vary with other parameter values.

Second, our analysis has assumed that platforms compete myopically by seeking a Nash equilibrium outcome in which profits cannot be increased in any period given the equilibrium strategy of the rival platform. In this case, strategic pricing decisions are assumed to depend only on their impact on profits during the current period. Network operators may alternatively choose prices by taking account of future profits, and in this case equilibrium results should depend on the present discounted value of future profits rather than current period profits.

Third, the behavior of content providers could be usefully generalized. In the present analysis, content is treated as a disposable commodity that must be renewed in every time period. More plausibly, content might be treated as a durable good in which content providers make investment decisions over several time periods. This generalization would significantly complicate the analysis, however. Whenever a content provider would choose to switch platforms, it should be allowed to take its investments in previous content with it. Individual content providers would therefore have, in principle, a different level of total content on each platform during every competitive period. Profit computations for providers and surplus computations for consumers would be significantly complicated, and in addition, the balanced traffic assumption used to simplify the calculations of platform profit might become increasingly unrealistic. In another generalization of the model, content providers could be allowed to “multi-home” by subscribing to both platforms instead of only one. We have not yet explored how this possibility would alter the results that we have obtained.

Finally, vertical integration between platforms and content providers is a potential issue that has not been modeled in the present analysis. Since many of the pricing scenarios which were examined – e.g. pricing policies toward content providers and blocking of off-platform content – were motivated by potential concerns over the evolution of NGN in the future, a formal analysis of the implications of vertical integration would be desirable.

8 References

Armstrong, M. (2006), "Competition in two-sided markets". *Rand Journal of Economics, 37,* 668-691.

Atkinson, J.M. and C.C. Barnekov (2000), "A Competitively Neutral Approach to Network Interconnection". OPP Working Paper No. 34, Federal Communications Commission.

Cohen, T. (2007). Next Generation Networks (NGN) Regulation Overview, Independent Communications Authority of South Africa: 38.

De Bijl, P. and M. Peitz. (2002), *Regulation and Entry into Telecommunications Markets*, Cambridge, UK: Cambridge University Press.

DeGraba, P. (2003), "Efficient Intercarrier Compensation for Competing Networks when Customers Share the Value of a Call". *Journal of Economics & Management Strategy,* Vol. 12, No. 2, 207-230.

Hermalin, B.E. and M. L. Katz (2006), "Customer or Complementor? Intercarrier Compensation with Two-Sided Benefits". Competition Policy Center Paper CPC06-060.

ITU (2007). "Trends in Telecommunication Reform 2007".

Jeon, D.-S., J.-J. Laffont and J. Tirole (2004), "On the 'receiver-pays' principle". *Rand Journal of Economics,* Vol. 35, No. 1, 85-110.

Laffont, J.-J., P. Rey and J. Tirole (1998a), "Network competition: I. Overview and nondiscriminatory pricing". *Rand Journal of Economics, 29,* 1-37.

Laffont, J.-J., P. Rey and J. Tirole (1998b), "Network competition: II. Price discrimination". *Rand Journal of Economics, 29,* 38-56.

Laffont, J.-J. and J. Tirole (2000), *Competition in Telecommunications,* Cambridge, MA: MIT Press.

Laffont, J.-J., S. Marcus, P. Rey and J. Tirole (2003), "Internet interconnection and the off-net pricing principle". *Rand Journal of Economics, 34,* 370-390.

Liebowitz, S. J. and S. E. Margolis. "Network Externalities (Effects)". (1999), http://www.utdallas.edu/~liebowit/palgrave/network.html

Marcus, S. (2006). "Framework for interconnection of IP-based networks - Accounting systems and interconnection regimes in the USA and the UK". Bad Honnef.

Rochet, J.-C. and J. Tirole (2003), "Platform competition in two-sided markets". *Journal of the European Economics Association,* Vol. 1, No. 4, 990-1209.

Schmalensee, R. and Evans, D. S. (2007) "Industrial Organization of Markets with Two-Sided Platforms". *Competition Policy International*, Vol. 3, No. 1.

Wu, T. (2003). "Network Neutrality, Broadband Discrimination". *Journal of Telecommunications and High Technology Law*, Vol. 2, p. 141.

Yoon, K. (2006). "Interconnection Economics of All-IP Networks". *Review of Network Economics 5(3)*, 351-365.

Appendix 1: Default Inputs

The following set of parameter values are used unless otherwise stated in the text of the paper.

Competitive Intensity	**Customer Demand**	**Marginal Costs**
$Z_C = 7500$	$aVoice = 10$	$c_{11} = c_{21} = 1$
$Z_P = 3000$	$bVoice = 0.02$	$c_{12} = c_{22} = 1$
Network Externality	$aData = 8$	$d_{11} = d_{21} = 2$
$ConsumerFixedUtility = 5000$	$bData = 0.02$	$d_{12} = d_{22} = 2$
$ProviderFixedUtility = 5000$	**Content Provider Supply**	**Fixed Charges**
$C_{X1} = 0.3$ $C_{X2} = 0.2$	$ProviderVariableCost = 500$	$f_C = 2000$
$P_{X1} = 0.2$ $P_{X2} = 0.1$	$ProviderFixedCost = 1000$	$f_P = 0$
Initial Market Shares	$AdRevMultiplier = 1$	**Access Charges**
$\alpha_1(0) = 0.9$	**Discount Rate per Period**	$ta_{1Voice} = ta_{2Voice} = 2$
$\beta_1(0) = 0.9$	$\delta = 0.976$	$ta_{1Data} = ta_{2Data} = 0$

In addition, our default input assumptions assume that subscription fees for content providers are feasible, and that usage fees for content providers are not feasible (therefore assumed to be equal to zero). In addition, content provided on either platform is assumed to be made freely available to subscribers on both platforms. All other input parameters consist of technical parameters such as starting values and scaling factors. These are available from the authors upon request.